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STABILITY FOR A CUBIC FUNCTIONAL EQUATIONS IN NON-ARCHIMEDEAN NORMED SPACES

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Abstract. In this paper, we investigate the functional equation

$$f(3x+y)+f(3x-y) = f(x+2y)+2f(x-y)+6f(2x)+3f(x)-6f(y)$$

and prove the generalized Hyers-Ulam stability for it in non-Archimedean normed spaces.

1. Introduction and preliminaries

S. M. Ulam [15] raised a question concerning the stability of functional equations in 1940 : *Let G_1 be a group and let G_2 be a meric group with the metric $d(\cdot,\cdot)$. Given $\epsilon > 0$, does there exist a $\delta > 0$ such that if a mapping $h : G_1 \longrightarrow G_2$ satisfies the inequality $d(h(xy), h(x)h(y)) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism $H : G_1 \longrightarrow G_2$ with $d(h(x), H(x)) < \epsilon$ for all $x \in G_1$?*

In 1941, Hyers [6] solved the Ulam problem for the case of approximately additive functions in Banach spaces. Since then, the stability of several functional equations have been extensively investigated by several mathematicians [2, 5, 7, 8]. Rassias [13], Jun and Kim [9] and Park and Jung [12] introduced the following functional equations

$$f(x+2y)+3f(x) = 3f(x+y)+f(x-y)+6f(y) \tag{1.1}$$

and

$$f(2x+y)+f(2x-y) = 2f(x+y)+2f(x-y)+12f(x) \tag{1.2}$$

and

$$f(3x+y)+f(3x-y) = 3f(x+y)+3f(x-y)+48f(x) \tag{1.3}$$

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and they established general solutions and the generalized Hyers-Ulam-Rassias stability problem for this functional equations, respectively. It is easy to see that the function $f(x) = cx^3$ is a solution of the functional equations (1.1), (1.2) and (1.3). Thus, it is natural that (1.1), (1.2) and (1.3) are called a *cubic functional equations* and every solution of the cubic functional equation is said to be a *cubic mapping*.

In this paper, we consider the following functional equation

$$\begin{aligned} &f(3x+y) + f(3x-y) \\ &= f(x+2y) + 2f(x-y) + 6f(2x) + 3f(x) - 6f(y). \end{aligned} \tag{1.4}$$

We prove the generalized Hyers-Ulam stability of (1.4) in complete non-Archimedean normed spaces.

A *valuation* is a function $|\cdot|$ from a field $\mathbb{K}$ into $[0, \infty)$ such that, for any $r, s \in \mathbb{K}$, the following conditions hold: (i) $|r| = 0$ if and only if $r = 0$, (ii) $|rs| = |r||s|$, (iii) $|r+s| \leq |r| + |s|$.

A field $\mathbb{K}$ is called *a valued field* if $\mathbb{K}$ carries a valuation. The usual absolute values of $\mathbb{R}$ and $\mathbb{C}$ are examples of valuations. If the triangle inequality is replaced by $|r+s| \leq max\{|r|, |s|\}$ for all $r, s \in \mathbb{K}$, then the valuation $|\cdot|$ is called a *non-Archimedean valuation* and a field with a non-Archimedean valuation is called *non-Archimedean field*. If $|\cdot|$ is a non-Archimedean valuation on $\mathbb{K}$, then clearly, $|1| = |-1|$ and $|n| \leq 1$ for all $n \in \mathbb{N}$.

Definition 1.1. Let X be a vector space over a scalar field $\mathbb{K}$ with a non-Archimedean nontrivial valuation $|\cdot|$. A function $\|\cdot\| : X \longrightarrow \mathbb{R}$ is called *a non-Archimedean norm (valuation)* if it satisfies the following conditions:

(a) $\|x\| = 0$ if and only if $x = 0$,
(b) $\|rx\| = |r|\|x\|$,
(c) the strong triangle inequality (ultrametric), that is,

$$\|x+y\| \leq max\{\|x\|, \|y\|\}$$

for all $x, y \in X$ and all $r \in \mathbb{K}$.

If $\|\cdot\|$ is a non-Archimedean norm, then $(X, \|\cdot\|)$ is called *a non-Archimedean normed space*.

Let $(X, \|\cdot\|)$ be a non-Archimedean normed space. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is said to be *convergent* if there exists an $x \in X$ such that $\lim_{n\to\infty} \|x_n - x\| = 0$. In that case, x is called *the limit of the sequence* $\{x_n\}$ and one denotes it by $\lim_{n\to\infty} x_n = x$. A sequence $\{x_n\}$ is said to be *Cauchy* in $(X, \|\cdot\|)$ if $\lim_{n\to\infty} \|x_{n+p} - x_n\| = 0$ for all

$p \in \mathbb{N}$. By (c) in Definition 1.1,

$$\|x_n - x_m\| \leq max\{\|x_{j+1} - x_j\| \mid m \leq j \leq n-1\} \quad (n > m)$$

and hence a sequence $\{x_n\}$ is Cauchy in $(X, \|\cdot\|)$ if and only if sequence $\{x_{n+1} - x_n\}$ converges to zero in $(X, \|\cdot\|)$. By *a complete non-Archimedean normed space* we mean one in which every Cauchy sequence is convergent.

Throughtout this paper, X is a non-Archimedean normed space and Y a complete non-Archimedean normed space.

2. The Generalized Hyers-Ulam stability for (1.4)

In 2003, Jun and Kim [10] introduced the following cubic functional equation

$$f(x+2y) + f(x-2y) + 6f(x) = 4f(x+y) + 4f(x-y) \tag{2.1}$$

and proved the generalized Hyers-Ulam stability for it in Banach spaces. In this section, we prove the generalized Hyers-Ulam stability of functional equation (1.4) in complete non-Archimedean normed spaces. We start the following theorem.

THEOREM 2.1. *Let $f : X \longrightarrow Y$ be a mapping. Then f satisfies (1.4) if and only if f is cubic.*

Proof. Suppose that f satisfies (1.4). Letting $x = y = 0$ in (1.4), we have $f(0) = 0$. Letting $y = 0$ in (1.4), we have

$$f(3x) - 3f(2x) - 3f(x) = 0 \tag{2.2}$$

for all $x \in X$ and letting $x = 0$ in (1.4) and relpacing y by x, we have

$$7f(x) - f(-x) - f(2x) = 0 \tag{2.3}$$

for all $x \in X$. Letting $y = x$ in (1.4), we have

$$f(4x) - f(3x) - 5f(2x) + 3f(x) = 0 \tag{2.4}$$

for all $x \in X$. By (2.2) and (2.4), we get

$$f(4x) = 2^3 f(2x) \tag{2.5}$$

for all $x \in X$. Repalcing $2x$ by x in (2.5), we get

$$f(2x) = 2^3 f(x) \tag{2.6}$$

for all $x \in X$. By (2.2) and (2.6), we get

$$f(3x) = 3^3 f(x) \tag{2.7}$$

for all $x \in X$. By (2.3) and (2.6), we get

$$f(-x) = -f(x) \tag{2.8}$$

for all $x \in X$. Replacing y by $3y$ in (1.4), by (2.6) and (2.7), we have

$$27f(x+y)+27f(x-y) = f(x+6y)+2f(x-3y)+51f(x)-6f(3y) \tag{2.9}$$

for all $x, y \in X$. Interchanging x and y in (2.9), by (2.8), we have

$$27f(x+y)-27f(x-y) = f(6x+y)-2f(3x-y)+51f(y)-6f(3x) \tag{2.10}$$

for all $x, y \in X$. Replacing y by $2y$ in (2.10), by (2.6), we have

$$\begin{aligned} &27f(x+2y) - 27f(x-2y) \\ &= 8f(3x+y) - 2f(3x-2y) + 408f(y) - 6f(3x) \end{aligned} \tag{2.11}$$

for all $x, y \in X$. Letting $y = -y$ in (2.11), by (2.8), we have

$$\begin{aligned} &27f(x-2y) - 27f(x+2y) \\ &= 8f(3x-y) - 2f(3x+2y) - 408f(y) - 6f(3x) \end{aligned} \tag{2.12}$$

for all $x, y \in X$. By (2.11) and (2.12), we get

$$8[f(3x+y)+f(3x-y)]-2[f(3x+2y)+f(3x-2y)]-12f(3x) = 0 \tag{2.13}$$

for all $x, y \in X$. Letting $x = \dfrac{x}{3}$ in (2.13), we have

$$f(x+2y) + f(x-2y) + 6f(x) = 4f(x+y) + 4f(x-y)$$

for all $x, y \in X$ and so f is additive-quadratic-cubic [10]. By (2.6), f is cubic. The converse is trivial. $\square$

For a given mapping $f : X \longrightarrow Y$, we define the difference operator $Df : X^2 \longrightarrow Y$ by

$$\begin{aligned} Df(x,y) &= f(3x+y) + f(3x-y) \\ &- f(x+2y) - 2f(x-y) - 6f(2x) - 3f(x) + 6f(y) \end{aligned}$$

for all $x, y \in X$.

THEOREM 2.2. *Let $\phi : X^2 \longrightarrow [0, \infty)$ be a mapping such that*

$$\lim_{n\to\infty} \frac{\phi(2^n x, 2^n y)}{|2|^{3n}} = 0 \tag{2.14}$$

for all $x, y \in X$ and let for each $x \in X$, the following limit

$$\begin{aligned} &\lim_{n\to\infty} max\left\{\left\{\frac{1}{|2|}\frac{\phi(2^{j-1}x, 0)}{|2|^{3(j-1)}} : 0 \le j < n\right\}\cup\right. \\ &\left.\left\{\frac{\phi(2^{j-1}x, 2^{j-1}x)}{|2|^{3(j-1)}} : 0 \le j < n\right\}\right\} \end{aligned} \tag{2.15}$$

denoted by $\widetilde{\phi}(x)$*, exist. Suppose that* $f : X \longrightarrow Y$ *is a mapping satisfying*

$$\|Df(x,y)\| \leq \phi(x,y) \tag{2.16}$$

for all $x, y \in X$*. Then there exists a cubic mapping* $C : X \longrightarrow Y$ *such that*

$$\|C(x) - f(x)\| \leq \frac{1}{|2|^6}\ \widetilde{\phi}(x) \tag{2.17}$$

for all $x \in X$*. In addition, if the limit*

$$\begin{aligned} &\lim_{i\to\infty}\lim_{n\to\infty} max\Big\{\Big\{\frac{1}{|2|}\frac{\phi(2^{j-1}x,0)}{|2|^{3(j-1)}} : i \leq j < n+i\Big\}\cup \\ &\Big\{\frac{\phi(2^{j-1}x,2^{j-1}x)}{|2|^{3(j-1)}} : i \leq j < n+i\Big\}\Big\} = 0 \end{aligned} \tag{2.18}$$

exists for all $x \in X$*, then* C *is the unique cubic mapping satisfying (2.17).*

Proof. Putting $x = y = 0$ in (2.16), we have

$$\|f(0)\| \leq \frac{1}{|2|^2}\phi(0,0)$$

and since $1 \leq \frac{1}{|2|}$, we get

$$\|f(0)\| \leq \frac{1}{|2|^3}\phi(0,0) \leq \frac{1}{|2|^{3n}}\phi(0,0)$$

for all $n \in \mathbb{N}$. By (2.14), $f(0) = 0$.

Putting $y = 0$ in (2.16), we have

$$\|f(3x) - 3f(2x) - 3f(x)\| \leq \frac{1}{|2|}\phi(x,0) \tag{2.19}$$

for all $x \in X$. Putting $y = x$ in (2.16), we have

$$\|f(4x) - f(3x) - 5f(2x) + 3f(x)\| \leq \phi(x,x) \tag{2.20}$$

for all $x \in X$. By (2.19) and (2.20), we get

$$\|f(4x) - 8f(2x)\| \leq max\Big\{\frac{1}{|2|}\phi(x,0), \phi(x,x)\Big\} \tag{2.21}$$

for all $x \in X$. Replacing x by $2^{n-1}x$ and dividing by $|2|^{3(n+1)}$ in (2.21), we get

$$
\begin{aligned}
&\Big\|\frac{f(2^{n+1}x)}{2^{3(n+1)}}-\frac{f(2^nx)}{2^{3n}}\Big\| \\
&\le \frac{1}{|2|^6}max\Big\{\frac{1}{|2|}\frac{\phi(2^{n-1}x,0)}{|2|^{3(n-1)}},\frac{\phi(2^{n-1}x,2^{n-1}x)}{|2|^{3(n-1)}}\Big\}
\end{aligned}
\tag{2.22}
$$

for all $x\in X$. By (2.14) and (2.22), we get $\left\{\frac{f(2^nx)}{2^{3n}}\right\}$ is Cauchy sequence. Since Y is complete, we conclude that $\left\{\frac{f(2^nx)}{2^{3n}}\right\}$ is convergent. Set

$$C(x):=\lim_{n\to\infty}\frac{f(2^nx)}{2^{3n}}.$$

Using induction one can show that

$$
\begin{aligned}
&\Big\|\frac{f(2^nx)}{2^{3n}}-f(x)\Big\|\le\frac{1}{|2|^6}max\Big\{\Big\{\frac{1}{|2|}\frac{\phi(2^{j-1}x,0)}{|2|^{3(j-1)}}:0\le j<n\Big\}\cup \\
&\Big\{\frac{\phi(2^{j-1}x,2^{j-1}x)}{|2|^{3(j-1)}}:0\le j<n\Big\}\Big\}
\end{aligned}
\tag{2.23}
$$

for all $n\in\mathbb{N}$ and all $x\in X$. By taking n to infinity in (2.23) and by (2.15), we obtain (2.17). Replacing x and y by 2^nx and 2^ny, respectively, and dividing by $|2|^{3n}$ in (2.16) and taking the limit as $n\to\infty$, by (2.14), we get

$$C(3x+y)+C(3x-y)=C(x+2y)+2C(x-y)+6C(2x)+3C(x)-6C(y)$$

for all $x,y\in X$. Therefore the mapping $C:X\longrightarrow Y$ satisfies (1.4) and so by Theorem 2.1, C is cubic.

Suppose that (2.18) holds. If C' is another cubic mapping satisfying (2.17), then by (2.18),

$$
\begin{aligned}
&\|C(x)-C'(x)\|=\lim_{i\to\infty}\frac{1}{|2|^{3i}}\|C(2^ix)-C'(2^ix)\| \\
&\le\lim_{i\to\infty}\frac{1}{|2|^{3i}}max\{\|C(2^ix)-f(2^ix)\|,\|f(2^ix)-C'(2^ix)\|\} \\
&\le\frac{1}{|2|^6}\lim_{i\to\infty}\lim_{n\to\infty}max\Big\{\Big\{\frac{1}{|2|}\frac{\phi(2^{j-1}x,0)}{|2|^{3(j-1)}}:i\le j<n+i\Big\}\cup \\
&\Big\{\frac{\phi(2^{j-1}x,2^{j-1}x)}{|2|^{3(j-1)}}:i\le j<n+i\Big\}\Big\}=0
\end{aligned}
$$

for all $x\in X$ and so $C=C'$. $\square$

From Theorem 2.2, we obtain the following corollary concerning the stability of (1.4).

COROLLARY 2.3. *Let* $\alpha_i : [0,\infty) \longrightarrow [0,\infty)$ $(i = 1,2,3)$ *be mappings satisfying*

(i) $\alpha_i(|2|) \neq 0$,
(ii) $\alpha_i(|2|t) \leq \alpha_i(|2|)\alpha_i(t)$ *for all* $t \geq 0$, *and*
(iii) $\alpha_1(|2|) < |2|^{\frac{3}{2}}, \alpha_2(|2|) < |2|^3$, *and* $\alpha_3(|2|) < |2|^3$.

Let $f : X \longrightarrow Y$ *be a mapping such that*

$$\|Df(x,y)\| \leq \delta[\alpha_1(\|x\|)\alpha_1(\|y\|) + \alpha_2(\|x\|) + \alpha_3(\|y\|)]$$

for all $x, y \in X$ *and some* $\delta > 0$. *Suppose that* $|2| < 1$. *Then there exists a unique cubic mapping* $C : X \longrightarrow Y$ *such that*

$$\|C(x) - f(x)\| \leq \frac{1}{|2|^6}\, \widetilde{\phi}(x)$$

for all $x \in X$, *where*

$$\widetilde{\phi}(x) = \delta|2|^2 max\Big\{\frac{\alpha_2(\|x\|)}{\alpha_2(|2|)}, |2|\Big[\Big(\frac{\alpha_1(\|x\|)}{\alpha_1(|2|)}\Big)^2 + \frac{\alpha_2(\|x\|)}{\alpha_2(|2|)} + \frac{\alpha_3(\|x\|)}{\alpha_3(|2|)}\Big]\Big\}.$$

Proof. Let $\phi(x,y) = \delta[\alpha_1(\|x\|)\alpha_1(\|y\|) + \alpha_2(\|x\|) + \alpha_3(\|y\|)]$. Then for any $n \in \mathbb{N}$

$$\begin{aligned}
&\frac{\phi(2^n x, 2^n y)}{|2|^{3n}} \\
&= \frac{\delta}{|2|^{3n}}\Big[\alpha_1(|2|^n\|x\|)\alpha_1(|2|^n\|y\|) + \alpha_2(|2|^n\|x\|) + \alpha_3(|2|^n\|y\|)\Big] \\
&\leq \delta\Big[\Big(\frac{(\alpha_1(|2|))^2}{|2|^3}\Big)^n \alpha_1(\|x\|)\alpha_1(\|y\|) + \Big(\frac{\alpha_2(|2|)}{|2|^3}\Big)^n \alpha_2(\|x\|) \\
&\quad + \Big(\frac{\alpha_3(|2|)}{|2|^3}\Big)^n \alpha_3(\|y\|)\Big]
\end{aligned}$$

for all $x, y \in X$. By (iii), we have

$$\lim_{n\to\infty} \frac{\phi(2^n x, 2^n y)}{|2|^{3n}} = 0$$

for all $x, y \in X$. Hence ϕ satisfies (2.14) in Theorem 2.2.
Let $x \in X$ and $j \in \mathbb{N} \cup \{0\}$. Then

$$\frac{1}{|2|}\frac{\phi(2^{j-1}x, 0)}{|2|^{3(j-1)}} \leq \frac{\delta}{|2|}\Big(\frac{\alpha_2(|2|)}{|2|^3}\Big)^{j-1}\alpha_2(\|x\|)$$

and

$$\begin{aligned}\frac{\phi(2^{j-1}x, 2^{j-1}x)}{|2|^{3(j-1)}} \leq\ & \delta\Big[\Big(\frac{(\alpha_1(|2|))^2}{|2|^3}\Big)^{j-1}(\alpha_1(\|x\|))^2 \\ & +\Big(\frac{\alpha_2(|2|)}{|2|^3}\Big)^{j-1}\alpha_2(\|x\|) + \Big(\frac{\alpha_3(|2|)}{|2|^3}\Big)^{j-1}\alpha_3(\|x\|)\Big]\end{aligned}$$

for all $x \in X$. By (iii), we obtain

$$\lim_{i\to\infty}\lim_{n\to\infty} max\Big\{\Big\{\frac{1}{|2|}\frac{\phi(2^{j-1}x, 0)}{|2|^{3(j-1)}} : i \leq j < n+i\Big\}\cup \Big\{\frac{\phi(2^{j-1}x, 2^{j-1}x)}{|2|^{3(j-1)}} : i \leq j < n+i\Big\}\Big\} = 0$$

for all $x \in X$ and so ϕ satisfies (2.18) in Theorem 2.2. Hence by Theorem 2.2, we have the result. □

EXAMPLE 2.4. *Let $\delta > 0$ and p be a real number with $p > \frac{3}{2}$. Suppose that $|2| < 1$. Let $f : X \longrightarrow Y$ is a mapping satisfying*

$$\|Df(x, y)\| \leq \delta(\|x\|^p\|y\|^p + \|x\|^{2p} + \|y\|^{2p})$$

for all $x, y \in X$. Then there exists a unique cubic mapping $C : X \longrightarrow Y$ satisfying (1.4) such that

$$\|C(x) - f(x)\| \leq \delta|2|^{-2(p+2)}\ max\{1, 3|2|\}\|x\|^{2p}$$

for all $x \in X$.

We have the following result which is analogous Theorem 2.2 for the functional equation (1.4).

THEOREM 2.5. *Let $\phi : X^2 \longrightarrow [0, \infty)$ be a mapping such that*

$$\lim_{n\to\infty} |2|^{3n}\phi\Big(\frac{x}{2^n}, \frac{y}{2^n}\Big) = 0$$

for all $x, y \in X$ and for each $x \in X$, and let for each $x \in X$, the following limit

$$\lim_{n\to\infty} max\Big\{\Big\{\frac{|2|^{3(j+2)}}{|2|}\phi\Big(\frac{x}{2^{j+2}}, 0\Big) : 0 \leq j < n\Big\}\cup \Big\{|2|^{3(j+2)}\phi\Big(\frac{x}{2^{j+2}}, \frac{x}{2^{j+2}}\Big) : 0 \leq j < n\Big\}\Big\}$$

denoted by $\phi_1(x)$, exist. Suppose that $f : X \longrightarrow Y$ is a mapping satisfying $f(0) = 0$ and

$$\|Df(x, y)\| \leq \phi(x, y)$$

for all $x, y \in X$. Then there exists a cubic mapping $C : X \longrightarrow Y$ satisfying (1.4) such that

$$\|C(x) - f(x)\| \leq \frac{1}{|2|^6}\ \phi_1(x) \tag{2.24}$$

for all $x \in X$. In addition, if the limit

$$\lim_{i\to\infty}\lim_{n\to\infty} max\left\{\left\{\frac{|2|^{3(j+2)}}{|2|}\phi\left(\frac{x}{2^{j+2}},0\right) : i \leq j < n+i\right\}\cup\right.$$

$$\left.\left\{|2|^{3(j+2)}\phi\left(\frac{x}{2^{j+2}},\frac{x}{2^{j+2}}\right) : i \leq j < n+i\right\}\right\} = 0,$$

then C is the unique cubic mapping satisfying (2.24).

The following corollary is an immediate consequence of Theorem 2.5.

Corollary 2.6. *Let $\alpha_i : [0,\infty) \longrightarrow [0,\infty)$ $(i = 1, 2, 3)$ be mappings satisfying*

(i) $\alpha_i\left(\frac{1}{|2|}\right) \neq 0$,
(ii) $\alpha_i\left(\frac{t}{|2|}\right) \leq \alpha_i\left(\frac{1}{|2|}\right)\alpha_i(t)$ *for all $t \geq 0$, and*
(iii) $\alpha_1\left(\frac{1}{|2|}\right) < \frac{1}{|2|^{\frac{3}{2}}}$, $\alpha_2\left(\frac{1}{|2|}\right) < \frac{1}{|2|^3}$, *and* $\alpha_3\left(\frac{1}{|2|}\right) < \frac{1}{|2|^3}$.

Let $f : X \longrightarrow Y$ be a mapping such that $f(0) = 0$ and

$$\|Df(x,y)\| \leq \delta[\alpha_1(\|x\|)\alpha_1(\|y\|) + \alpha_2(\|x\|) + \alpha_3(\|y\|)]$$

for all $x, y \in X$ and some $\delta > 0$. Then there exists a unique cubic mapping $C : X \longrightarrow Y$ such that

$$\|C(x) - f(x)\| \leq \frac{1}{|2|^6}\ \phi_1(x)$$

for all $x \in X$, where

$$\phi_1(x) = \delta|2|^6 max\left\{\frac{1}{|2|}\left(\alpha_2\left(\frac{1}{|2|}\right)\right)^2\alpha_2(\|x\|),\ \left(\alpha_1\left(\frac{1}{|2|}\right)\right)^4(\alpha_1(\|x\|))^2 + \right.$$

$$\left.\left(\alpha_2\left(\frac{1}{|2|}\right)\right)^2\alpha_2(\|x\|) + \left(\alpha_3\left(\frac{1}{|2|}\right)\right)^2\alpha_3(\|x\|)\right\}.$$

Example 2.7. *Let $\delta > 0$ and p be a real number with $p < \frac{3}{2}$. Suppose that $|2| < 1$. Let $f : X \longrightarrow Y$ is a mapping satisfying $f(0) = 0$ and*

$$\|Df(x,y)\| \leq \delta(\|x\|^p\|y\|^p + \|x\|^{2p} + \|y\|^{2p})$$

for all $x, y \in X$. Then there exists a unique cubic mapping $C : X \longrightarrow Y$ satisfying (1.4) such that

$$\|C(x) - f(x)\| \leq \delta|2|^{-(4p+1)} max\Big\{1, 3|2|\Big\}\|x\|^{2p}$$

for all $x \in X$.

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